4.6 Honors
Solving Quadratic Equations by Completing the Square

Objective: Solve quadratic equations by completing the square.

Can we solve $x^2+8x=7$ using the methods we know?

Completing the Square

In Lesson 4.3 you learned that to make the expression x^2+bx a perfect square, you must add $\left(\frac{b}{2}\right)^2$ to the expression. Every perfect square trinomial can be written as a binomial squared.

Perfect Square Trinomial	Binomial Squared
$x^2+bx+\left(\frac{b}{2}\right)^2$	$\left(x+\frac{b}{2}\right)^2$

The method of completing the square can be used to solve any quadratic equation. When you complete the square as part of solving an equation, you must add the same number to both sides of the equation.

Change to decimals $-.697, -4.302$

Examples

Solve each quadratic equation by completing the square.

1. $x^2+8x=7$

$b=8$

$\left(\frac{8}{2}\right)^2$

$x^2+8x+16=7+16$

$(x+4)^2=23$

$x+4=\pm\sqrt{23}$

$x=-4\pm\sqrt{23}$

2. $x^2-6x+11=0$

$x^2-6x=-11$

$x^2-6x+9=-11+9$

$(x-3)^2=-2$

$x-3=\pm\sqrt{-2}$

$x=3\pm i\sqrt{2}$

3. $x^2+5x+3=0$

$x^2+5x=-3$

$x^2+5x+\frac{25}{4}=-3+\frac{25}{4}$

$\left(x+\frac{5}{2}\right)^2=\frac{13}{4}$

$x+\frac{5}{2}=\pm\sqrt{\frac{13}{4}}$

$x=-\frac{5}{2}\pm\frac{\sqrt{13}}{2}$

$x=\frac{-5\pm\sqrt{13}}{2}$

4. $\frac{2x^2-24x+8=0}{2\quad 2}$

$x^2-12x+4=0$

$x^2-12x=-4$

$x^2-12x+36=-4+36$

$(x-6)^2=32$

$x-6=\pm\sqrt{32}$

$x=6\pm\sqrt{32}$

$x=6\pm4\sqrt{2}$

5. $\frac{-3x^2-6x-9=0}{-3\quad -3}$

$x^2+2x+3=0$

$x^2+2x=-3$

$x^2+2x+1=-3+1$

$(x+1)^2=-2$

$x+1=\pm\sqrt{-2}$

$x=-1\pm\sqrt{-2}$

$x=-1\pm i\sqrt{2}$

6. $x^2-8x-9=0$

$x^2-8x=9$

$x^2-8x+16=9+16$

$(x-4)^2=25$

$x-4=\pm\sqrt{25}$

$x=4\pm\sqrt{25}$

$x=4\pm5$

$x=9,-1$

Solve by factoring: $x^2-8x-9=0$

$(x-9)(x+1)=0$

$x=9,-1$