

4.3 Honors

Converting from Standard Form to Vertex Form

Objective: Convert quadratic equations from standard form to vertex form by completing the square.

Completing the Square

To make the expression $x^2 + bx$ a perfect square trinomial, you must add $\left(\frac{b}{2}\right)^2$ to the expression.

Perfect Square Trinomial

$$\underline{x^2 + bx + \left(\frac{b}{2}\right)^2}$$

Binomial Squared

$$\underline{\left(x + \frac{b}{2}\right)^2}$$

↖ equivalent expressions ↗

Examples

Find the number that must be added to each expression to form a perfect square trinomial. Then write the trinomial as a binomial squared.

1. $x^2 + 16x + \underline{64}$

$$(x + 8)^2$$

2. $x^2 - 8x + \underline{16}$

$$(x - 4)^2$$

3. $x^2 + 5x + \underline{\frac{25}{4}}$

$$\left(x + \frac{5}{2}\right)^2$$

Rewrite each equation in vertex form. Then identify the vertex.

4. $f(x) = (x^2 + 8x) + 3$

$$\left(\frac{8}{2}\right)^2 = (x^2 + 8x + 16) + 3 - 16$$

$$\left(\frac{8}{2}\right)^2 = (x + 4)^2 - 13$$

Vertex $(-4, -13)$

5. $y = (x^2 - 4x) - 7$

$$= (x^2 - 4x + 4) - 7 - 4$$

$$= (x - 2)^2 - 11$$

Vertex $(2, -11)$

6. $f(x) = (x^2 + 5x) + 2$

$$= (x^2 + 5x + \frac{25}{4}) + 2 - \frac{25}{4}$$

$$= \left(x + \frac{5}{2}\right)^2 - \frac{17}{4}$$

Vertex $\left(-\frac{5}{2}, -\frac{17}{4}\right)$

$$7. f(x) = (x^2 + 7x) - 3$$

$$= (x^2 + 7x + \frac{49}{4}) - 3 - \frac{49}{4}$$

$$= (x + \frac{7}{2})^2 - \frac{61}{4}$$

vertex $(-\frac{7}{2}, -\frac{61}{4})$

$$8. f(x) = (2x^2 + 12x) + 25$$

$$= 2(x^2 + 6x) + 25$$

$$= 2(x^2 + 6x + 9) + 25 - 18$$

$$= 2(x + 3)^2 + 7$$

vertex $(-3, 7)$

$$9. f(x) = (3x^2 + 9x) - 2$$

$$= 3(x^2 + 3x) - 2$$

$$= 3(x^2 + 3x + \frac{9}{4}) - 2 - \frac{27}{4}$$

$$= 3(x + \frac{3}{2})^2 - \frac{35}{4}$$

vertex $(-\frac{3}{2}, -\frac{35}{4})$

10. A high school is planning a community carnival to raise money for a local charity. Based on data they have collected from similar events held in the area, they believe that they can model the expected attendance by the function $f(x) = -0.5x^2 + 80x - 2425$.

where x represents the high temperature for the day, in °F

a. Write the function in vertex form by completing the square.


$$f(x) = (-0.5x^2 + 80x) - 2425$$

$$= -0.5(x^2 - 160x) - 2425$$

$$= -0.5(x^2 - 160x + 6400) - 2425 + 3200$$

$$= -0.5(x - 80)^2 + 775$$

b. What is the vertex? Is the vertex a maximum or a minimum?

 $(80, 775)$ max

c. Interpret the vertex in the context of the problem.

If the high temperature for the day is 80°F , 775 people can be expected to attend.