2-5 Parallel Lines and Triangles

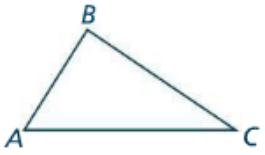
G.CO.10 Prove theorems about triangles . . . measures of interior angles of a triangle sum to 180°. Also G.CO.9

Objectives To use parallel lines to prove a theorem about triangles
To find measures of angles of triangles

take note

Theorem 16 Triangle Angle-Sum Theorem

The sum of the measures of the angles of a triangle is 180.



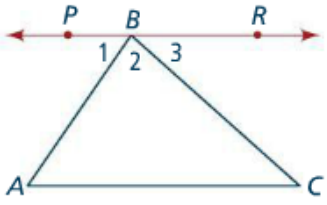
$m\angle A + m\angle B + m\angle C = 180$

The proof of the Triangle Angle-Sum Theorem requires an *auxiliary line*. An **auxiliary line** is a line that you add to a diagram to help explain relationships in proofs. The red line in the diagram below is an auxiliary line.

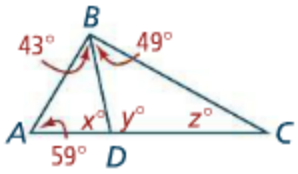
Proof Proof of Theorem 16: Triangle Angle-Sum Theorem

Given: $\triangle ABC$

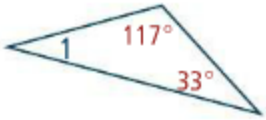
Prove: $m\angle A + m\angle 2 + m\angle C = 180$



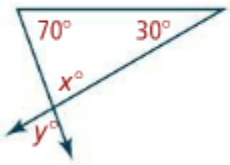
Got It? Use the diagram below. What is the value of z?



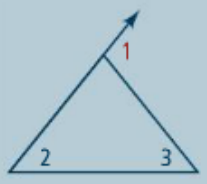
Practice 1. Find $m\angle 1$.

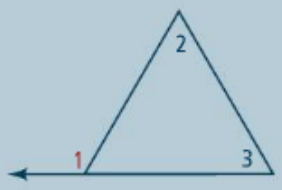


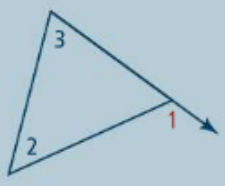
2. **Algebra** Find the value of each variable.



An **exterior angle of a polygon** is an angle formed by a side and an extension of an adjacent side. For each exterior angle of a triangle, the two nonadjacent interior angles are its **remote interior angles**. In each triangle below, $\angle 1$ is an exterior angle and $\angle 2$ and $\angle 3$ are its remote interior angles.





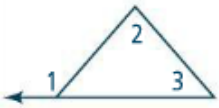


The theorem below states the relationship between an exterior angle and its two remote interior angles.

take note

Theorem 17 Triangle Exterior Angle Theorem

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

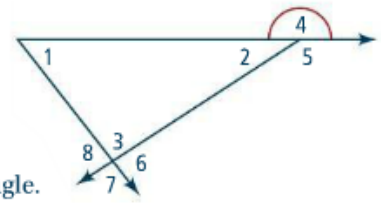


$$m\angle 1 = m\angle 2 + m\angle 3$$

You will prove Theorem 17 in Exercise 26.

You can use the Triangle Exterior Angle Theorem to find angle measures.

Practice **3. a.** Which of the numbered angles are exterior angles?

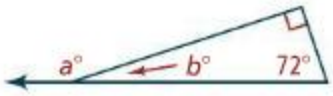


b. Name the remote interior angles for each exterior angle.

c. How are exterior angles 6 and 8 related?

4. Algebra Find the measure of $\angle 2$.

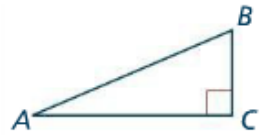
5. A ramp forms the angles shown at the right. What are the values of a and b?



6. A lounge chair has different settings that change the angles formed by its parts. Suppose $m\angle 2 = 71$ and $m\angle 3 = 43$. Find $m\angle 1$.

Homework

Proof **19.** Prove the following theorem: The acute angles of a right triangle are complementary.

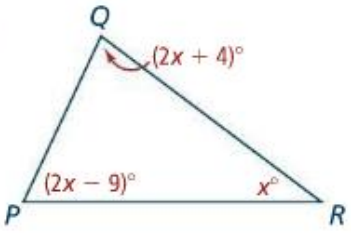


Given: $\triangle ABC$ with right angle C

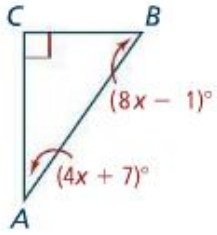
Prove: $\angle A$ and $\angle B$ are complementary.

Find the values of the variables and the measures of the angles.

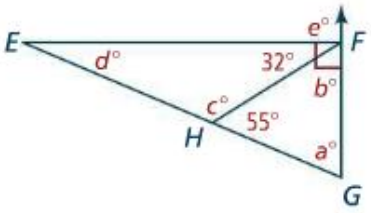
22.



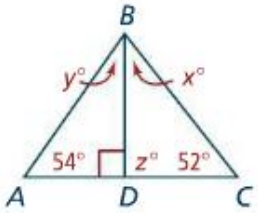
23.



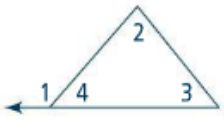
24.



25.



Proof **26.** Prove the Triangle Exterior Angle Theorem (Theorem 17). The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.



Given: $\angle 1$ is an exterior angle of the triangle.

Prove: $m\angle 1 = m\angle 2 + m\angle 3$

27. Reasoning Two angles of a triangle measure 64 and 48. What is the measure of the largest exterior angle of the triangle? Explain.

34. In the figure at the right, $\overline{CD} \perp \overline{AB}$ and $\overline{CD}$ bisects $\angle ACB$. Find $m\angle DBF$.